What a Drag!: Instructor Guide

Title

What a Drag!

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Discipline

Physics and Astronomy

Target Audience

Introductory, science and engineering majors

Keywords

Air resistance, drag, force, graphing, projectile, trajectory

Length of Time/Staging

Two class periods

Abstract

Students explore the effects of air resistance on the path of a baseball. Forces acting on a baseball are identified and the resulting set of differential equations for both the horizontal and vertical motion is derived. A computer program for solving these equations for the trajectory of a spherical object is provided. Students use the program to examine realistic trajectories of baseballs (or other spherical objects). Additional activities include determining at what angle one

gets the maximum range when air resistance is not negligible and whether baseballs travel farther in high altitude parks.

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Format of Delivery

The problem can be given as a homework assignment to an individual or a small group of two or three. Stages are given separately over two lecture periods. Reserve 10-15 minutes of lecture time after presenting part 1 for students to discuss the problem. Most of the work for part 2 is done outside of class. Prior to delivery of part 2, a mini-lecture on aerodynamic drag and numerical computation should help students transition to using a computer program to find answers empirically. The Teaching Notes include some material that can be part of the mini-lecture. A wrap-up could include presentations by some individuals/groups during lecture time.

Student Learning Objectives

Basic science:

Newton's second law is used to determine the equation of motion of a baseball in the presence of air resistance. The aerodynamic drag force is identified and its dependence on the physical properties of the object, air, and velocity is investigated. The resultant equation of motion is solved numerically using a computer program. The program enables students to gather data on trajectories under varying initial conditions, thereby allowing them to experience the empirical side of science. Other objectives include practice in plotting trajectories, velocities, and acceleration, and interpreting graphical data.

Real world context:

Physics is applied to realistic projectile motion. Students find firsthand the extent to which neglecting the effect of air resistance can be a poor approximation. Numerical computation is introduced as an important tool for solving practical problems and for discovery. Note: this problem was not designed with the intent of having students learn the underlying physics of viscosity nor about numerical methods for solving differential equations.

Student Resources

Object	Mass	Diameter	Frontal Area
	(kg)	*(m)*	*(m^2)*
Baseball	0.1453	0.0738	0.004275

Golf	0.046	0.042	0.001385
Ping-pong	0.003	0.040	0.001257
Tennis ball	0.056	0.060	0.002827

Homerun balls typically have post-impact speeds of ~ 100 mph/hr or ~ 45 m/s.

The depth of a ballpark's outfield can vary greatly within the outfield and among different ballparks. In the Phillies' new ballpark (Citizens Bank Park), the straightaway center is at 401 ft with a 6-foot wall.

Altitude data (above sea level):

- Mexico City, Mexico: 2240 m
- Monterrey, Mexico: 540 m
- Philadelphia, PA USA: 11.6 m

Popular Press

1. "The myth of the 500-foot homerun: Do men always exaggerate the length of their long balls?"

   http://www.slate.com/articles/news_and_politics/hey_wait_a_minute/1997/10/the_myth_of_the_500foot_home_run.html

2. "Can a baseball be hit farther at high altitude?" in the The Straight Dope

   www.straightdope.com/mailbag/mhighaltbaseball.html

Baseball and Sports Organizations

1. Major League Baseball
2. Philadelphia Phillies

   Phillies' new ballpark:

   http://philadelphia.phillies.mlb.com/phi/ballpark/information/index.jsp?content=facts_figures

Physics of Baseball

1. Prof. Alan Nathan: "The Physics of Baseball"
2. Exploratorium: "Science of Baseball"

Physics of Baseball Books

1. *The Physics of Baseball*, by Robert K. Adair
2. *Keep Your Eye on the Ball: The Science and Folklore of Baseball* by Robert G. Watts, A. Terry Bahill (Contributor)

Selected Published Research on the Physics of Baseball Bats

1. Brody, H. (1986). "The sweet spot of a baseball bat." *American Journal of Physics*, 54(7), 640-643.

2. Cross, R. (1998). "The sweet spot of a baseball bat." *American Journal of Physics*, 66(9), 771-779.
3. Nathan, A.M. (2000). "The dynamics of the baseball-bat collision." *American Journal of Physics*, 68(11), 979-990.
4. Greenwald, R.M., Penna, L.H., and Crisco, J.J. (2001). "Difference in batted ball speed with wood and aluminum baseball bats." *Journal of Applied Biomechanics*, 17, 241.

Integration and Numerical Solutions of Differential Equations

1. Numerical recipe books online (www.nr.com). Chapters 16 and 4 are relevant.
2. *Computational Physics* by Nicholas J. Giordano

Instructor Resources

The source code for trajectory.exe is in the problem folder.

Executable file: trajectory.exe

The program has two text-based menus for inputting parameters. The basic menu is where the altitude, post-impact velocity, angle, and time step can be entered. The time step is used in the numerical integration of the set of differential equations; it also determines the time interval between data points in the output file. Values ranging from 0.1 and 0.02 yield between 50 and 250 data points for a typical trajectory. The computational time is hardly noticeable on modern PCs (>500 MHz). There is also an input to switch off the drag force so that ideal (in-vacuum) trajectories can be generated for comparison. The advanced menu allows one to change the mass, diameter, and frontal area of the spherical object. Use this to investigate the trajectories of other objects (tennis balls, golf balls, etc.). The default values for the advanced menu are the parameters of a baseball. Changing the values of the advanced input parameters carries through to the basic menu. These parameters are set to those of the baseball when the program is initially executed.

The output file is saved in the same directory as where the executable resides. This text file consists of seven tab-delimited columns corresponding to time and the horizontal and vertical components of the position, velocity, and acceleration of the object. A single header line starts the output file and identifies the content of each column, namely: time, x, y, vx, vy, ax, and ay. The data file terminates when the ball descends to its initial altitude. Before the data file is closed an additional line "End of Baseball." The values for the vertical position are all referenced to the initial altitude; *i.e.* all trajectories start and end at y=0.

trajectory.exe is based on Ralph Carmichael's Fortran program that is available from Public Domain Aeronautical Software atwww.pdas.com/bb2.htm. The following information is derived from this website and may be helpful if you decide to modify our C++ code.

Aerodynamic drag of a sphere:

The drag coefficient of a sphere traveling at speeds typical of baseballs is about 0.5, but a procedure taken from Chow, An Introduction to Computational Fluid Mechanics, is included to let you use this program to study other spheres. To use this routine, the Reynolds number of the flow must be computed, which requires the density of the air as well as the viscosity of air.

Routines in the program compute these quantities and eventually return the drag coefficient to the procedure that computes the acceleration.

Numerical integration of the differential equations:

Fixed step size, fourth-order Runge-Kutta algorithm. Some background material on Runge-Kutta can be found in Numerical Recipes.

Correction of the final point:

As the numerical integration advances in time, the fixed step size in time will most likely step over the point where the altitude is exactly equal to the initial altitude and give a point with negative altitude as the final point. To make the solution look good, an interpolation is done in the final interval in order to land right on zero altitude.

Assumption of zero spin:

The analysis above assumes that there is negligible spin on the ball and the total aerodynamic force is exactly opposite in direction to the velocity.

Author's Teaching Notes

The problem can be introduced during a section on kinematics shortly after Newton's second law (forces) is introduced. Students should be familiar with trajectories of projectiles under the usual assumptions that air resistance is negligible and gravity is the only force acting on the object. The problem can be easily modified for non-calculus based courses by dropping references to differential equations.

Part 1 is relatively quick with the help of the Student Resources, but students should be encouraged to produce a one-page write up to avoid trivial answers.

Part 2 benefits from a mini-lecture on aerodynamic drag force and numerical solutions. Below are some notes that serve as a guide to the content of the mini-lecture.

Output files are in tab-delimited, ASCII format. Students can use Excel or other spreadsheet software with graphing capabilities to plot trajectories.

Aerodynamic drag force

Please see aerodynamic drag force.pdf in the problem folder.

Numerical methods for solving differential equations

Please see numerical methods for differential equations.pdf in the problem folder.

Assessment Strategies

The logic and content of the task force report can be used to judge whether its conclusions/recommendations are supported by quantitative results. To mitigate grading complaints, make clear any expectations of what the report should include. Students that are inexperienced with PBL may find it helpful to refer to an outline that lists material that should be included and discussed in the report. Test or quiz questions on the additional activities listed in part 2 also work well.

Solution Notes

Part 1

1. The two main forces are gravity (directed vertically down) and air drag. If the ball is spinning, there is also a magnus force that is directed perpendicular to the both the axis of rotation and velocity. Of course, forces due to wind may also be present.
2. Air drag is opposite to the direction of the velocity, i.e., tangent to any point on the trajectory. The magnitude of the drag force depends on the mass, size, shape, texture, and speed of the object. It also depends on the density of the air and its viscosity. These are functions of temperature, pressure, and even humidity. Interestingly, humid air is less dense and so provides less drag force. At high altitudes the air is cooler and less dense than at sea level.
3. The flow of air around objects the size of baseballs and traveling at the speeds of baseballs is often described as turbulent. The aerodynamic drag force increases as the square of the velocity, as compared to laminar flow where it is linear in velocity. Assuming the linear term is negligible and that there is no magnus force nor wind, the differential equation of motion can be written as:

   $$m\frac{d\vec{v}}{dt} = m\vec{g} - \frac{1}{2}CA\rho v^2 \frac{\vec{v}}{|\vec{v}|}$$

   where the second term is the drag force.

   This expression yields

   horizontal: $$mv_x = -\frac{1}{2}CA\rho v^2 \cos\theta$$

   vertical: $$mv_y = -mg - \frac{1}{2}CA\rho v^2 \sin\theta$$

4. The nonlinear dependence of the drag force on velocity complicates finding an analytic solution.

Part 2 of the solution guide, containing the graphs for the problem, is attached in the problem folder.

Acknowledgements

We basically re-engineered Ralph L. Carmichael's Fortran program, which is available from Public Domain Aeronautical Software (www.pdas.com). His Fortran program was translated into C++ and a user interface was added so that multiple initial conditions could be run without recompilation. We kept the interface simple in order to maximize portability of the program. The software redevelopment was largely done by Maitreya Natu (Ed Nowak and Paul Hyde helped a bit) and was supported in part by a grant from ITUE/PRESENT at the University of Delaware. Edmund R. Nowak is a Cottrell Scholar of the Research Corporation.

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