What a Drag!: Problem Handouts

What a Drag!

Part 1

Major League Baseball (MLB) is looking to sell the Montreal Expos, which it currently owns. Competitive bids have been submitted by two Mexican cities, Mexico City and Monterrey. Prior to making a decision, MLB has asked your consulting firm to evaluate the effect that altitude would have on a fly ball in these two baseball stadiums. The MLB is interested in preserving its historical home run records and maintaining the offensive-defensive balance of the game. In particular, they want you to determine how much further baseballs will travel in these two stadiums compared to United States stadiums, most of which are near sea level.

Consider the following questions:

1. What are the forces acting on a baseball as it moves through air? Sketch a force diagram at various points along a hypothetical trajectory.
2. In which direction does the force of air resistance, or aerodynamic drag force, act on the baseball? What sorts of factors (*e.g.* properties of air and the baseball) does the force of air resistance depend upon? How does altitude work its way into the drag force?
3. After choosing an appropriate mathematical form to describe the aerodynamic drag force, write down the differential equation of motion for the baseball and decompose it into its horizontal and vertical components.
4. Solving these equations for the special case of no drag force is easy and leads to the familiar range equation. What makes it difficult to find an analytic solution when the drag force is included?

What a Drag!

Part 2

Nearly all interesting science problems and engineering challenges involve nonlinear differential equations for which analytical solutions often do not exist. In these instances, solutions must be found numerically with the aid of computers. Trajectory.exe is a program that solves the equations of motion of a spherical object moving through still air. The aerodynamic drag force is exactly opposite in direction to the object's velocity and is assumed to have a magnitude given by:

$$\frac{1}{2}C\rho A v^2$$

where ρ is the density of air, C is the drag coefficient, A is the frontal cross-sectional area, and v is the velocity of the object. This is a good approximation for objects around which the air flows in a turbulent manner,*e.g.*, baseballs, tennis balls, golf balls. Altitude's main effect is on ρ; the density of air decreases with altitude. The drag coefficient actually depends on the shape and texture of the object's surface, but this program assumes a smooth sphere and calculates the drag coefficient based on its cross-sectional area, and the density and viscosity of the air. The program also assumes there is negligible spin on the ball and hence no magnus force. A batted ball can have significant spin, but this is due to a glancing hit for which the post-impact velocity will be low. For a ball to be hit far, it must be hit squarely.

Use this program to obtain data about trajectories of baseballs hit in stadiums having altitudes corresponding to those of Mexico City, Monterrey, and sea level. As part of your analysis you should:

1. Determine the optimal angle for maximizing the range in the presence of air resistance.
2. Examine the percentage increase (or decrease) in range compared to sea level for a number of realistic post-impact velocities.

Compose a written report for Major League Baseball that details your findings. Your report should include data tables and graphs of pertinent information to support your conclusions.

Additional effects to explore include:

1. Plot trajectories with and without air resistance for the same initial conditions. How do they differ? Explain the asymmetrical shape of trajectories when air resistance is not negligible.
2. What is the optimal angle for longest range for the cases of air resistance and no air resistance?
3. A ping-pong ball and golf ball are nearly the same size. Examine their respective trajectories for the same initial conditions. What accounts for the difference in range? (Use the advanced menu to input the diameter, mass, and frontal area of each ball.)
4. What is the minimum post-impact velocity necessary to clear the deepest part of the outfield in your favorite ballpark? Don't have a favorite ballpark? Consider the

Philadelphia Phillies' new ballpark. Remember to take into account the height of the outfield wall.